30
Maths
Starters

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Introduction

This is a collection of puzzles for Maths lessons. Hopefully you'll find these are a bit more interesting than the "Let's count up in 0.2's" type of lesson starter that teachers in England are encouraged to use. It is also hoped that some of these puzzles can be used as a talking point to introduce new mathematical ideas.

There are plenty of old chestnuts here, along with some new puzzles. There are also a few generic starters for when you're feeling lazy...

Suggested uses:

- Print onto transparencies for lesson starters
- Print and laminate for extension or small group works
- Duplicate two to a page for homework

The star rating is a rough guide to difficulty:

✰	Little teacher input is required. These puzzles don't require much mathematics beyond basic arithmetic.
✰✰	These may require explanations of key words by a teacher. They may require more work than one star puzzles, or a particular insight to solve them.
✰✰✰	These puzzles can be difficult, and may need a lot of teacher input.

Find The Path ☆ 1

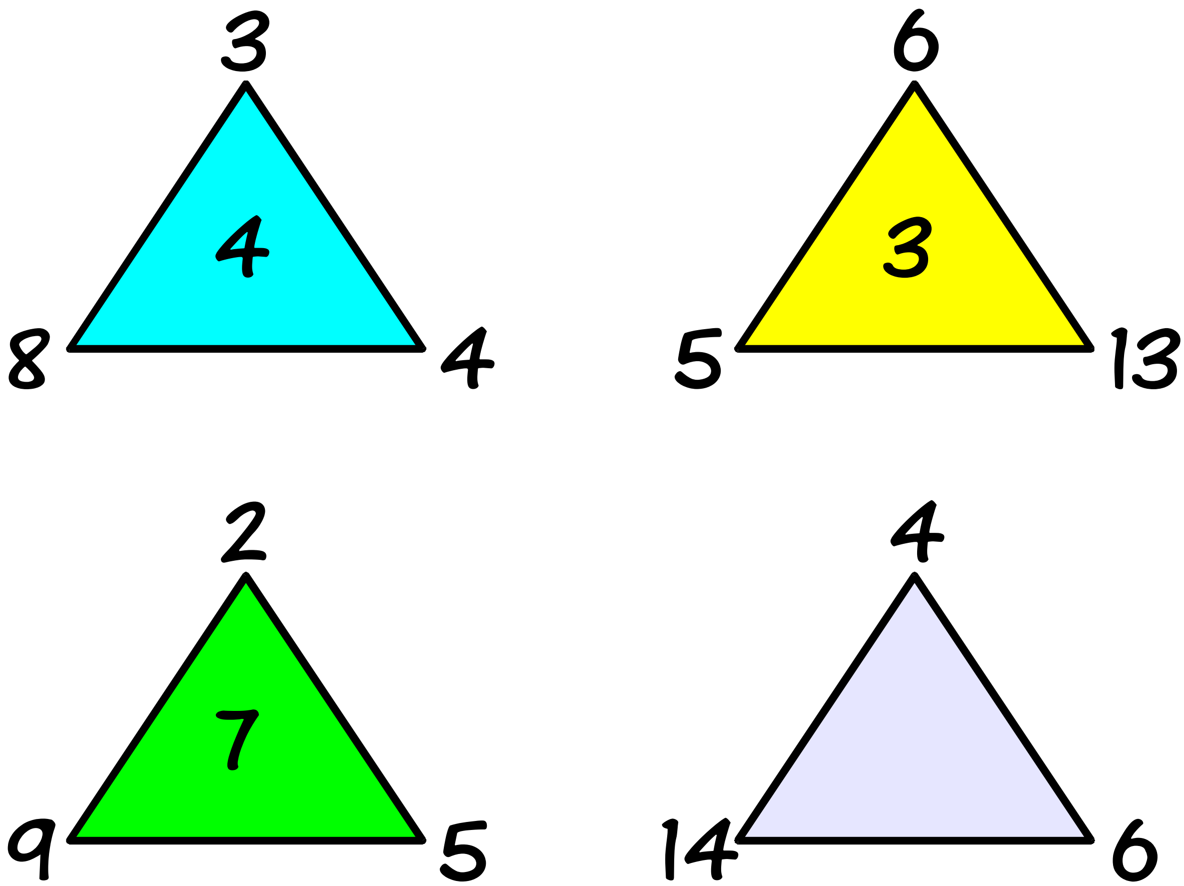
Start at the bottom left square and move up, down, left or right until you reach the finish.

	4	9	7	7	4 ⇨ Finish
	8	9	4	5	7
	6	6	4	9	9
	7	8	8	8	6
Start ⇨	5	5	6	5	5

Add the numbers as you go. Can you make exactly 53 ?

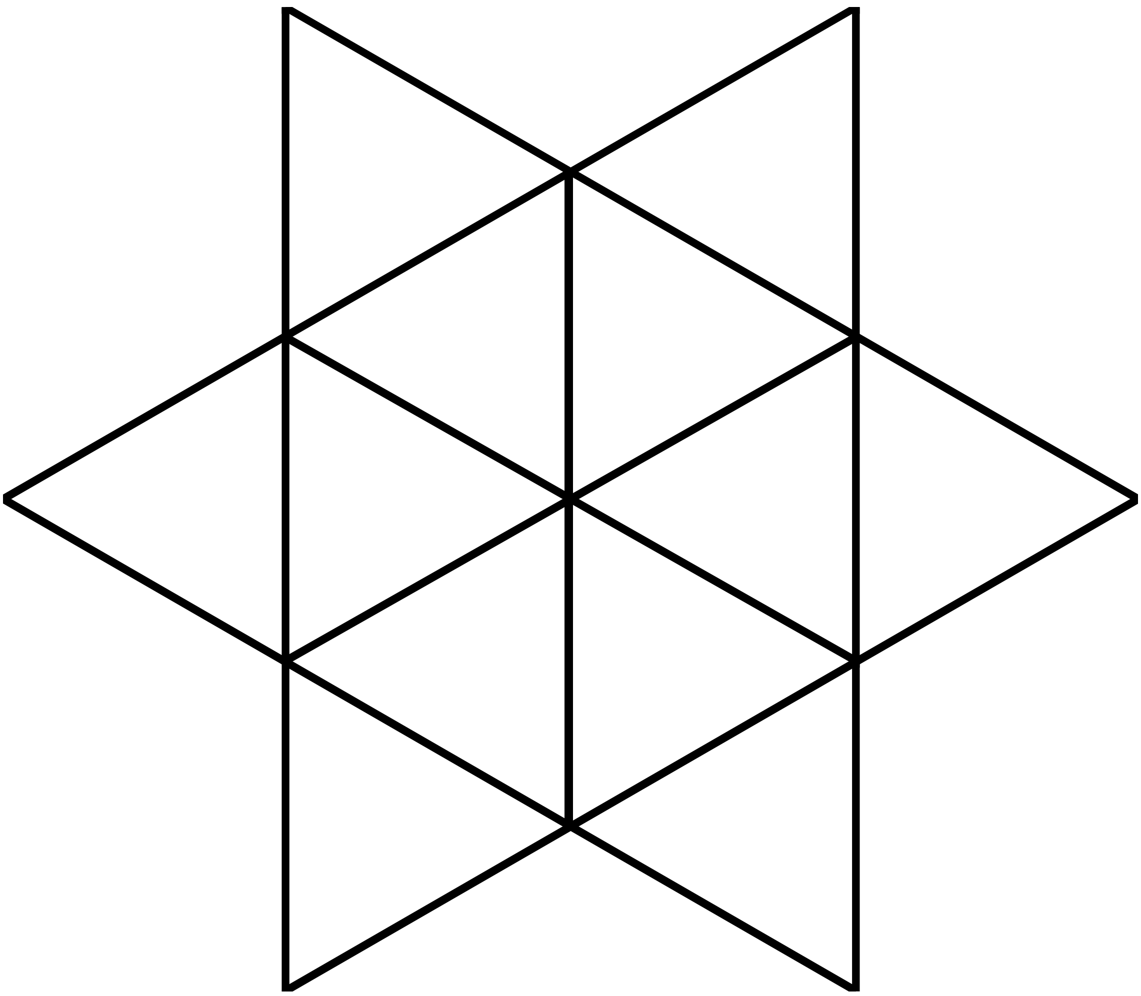
Missing Number ☆ 2

Which number should go in the empty triangle?



How Many Triangles? ☆ 3

How many triangles can you see in this picture?

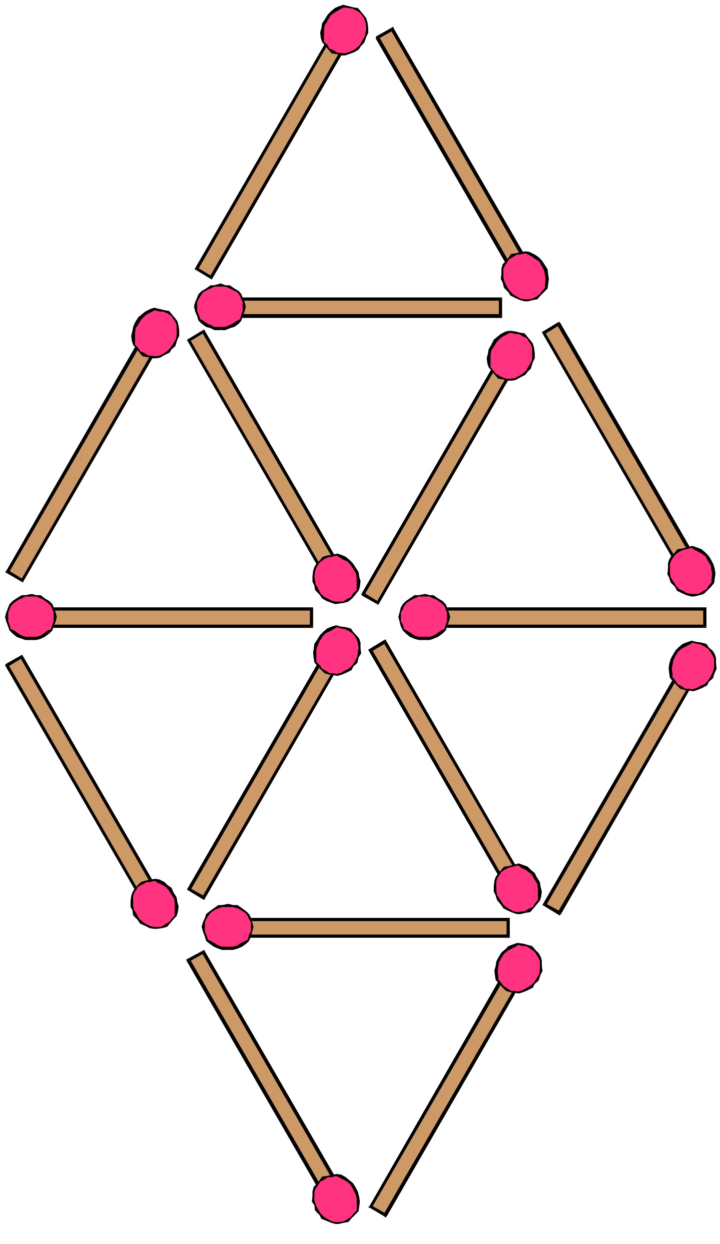


There is something strange about this addition square. Can you work out what the missing number is?

+	3	8	11
3	6	11	2
8	11	4	7
11	2	7	

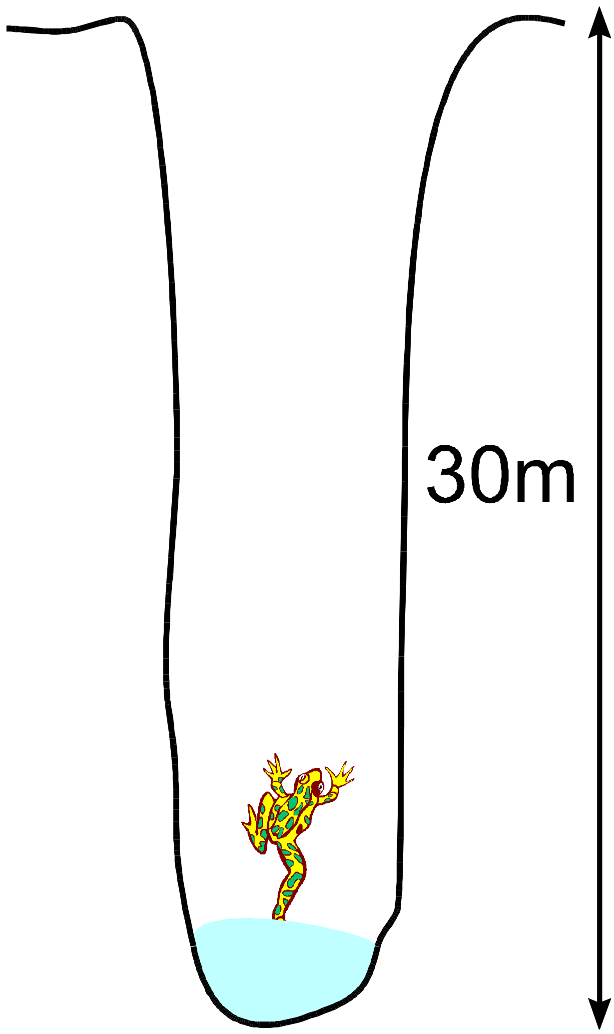
Missing Matches ☆ 5

Remove just 4 matches to leave 4 equilateral triangles – they must be all the same size.



The Jumping Frog ☆ 6

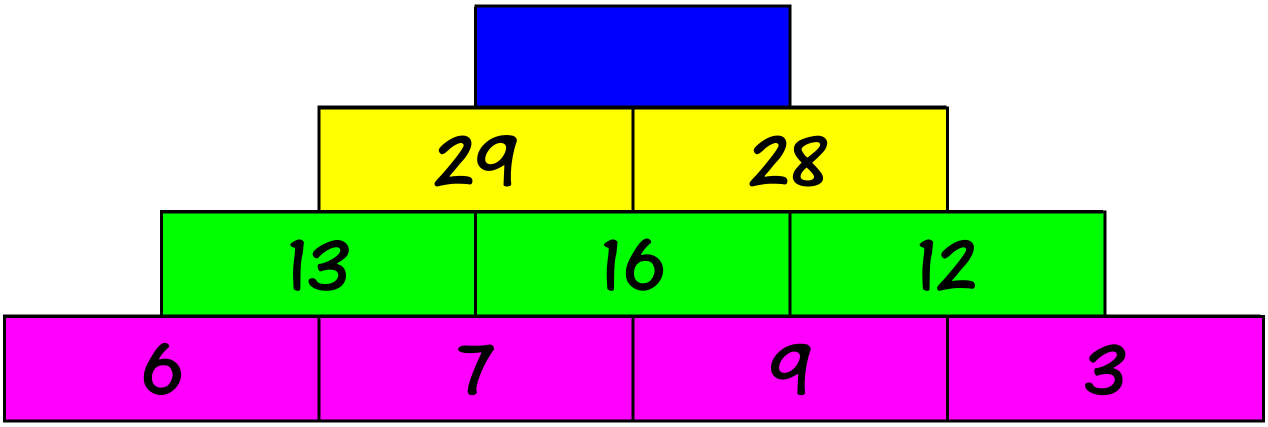
A frog has fallen into a pit that is 30m deep.



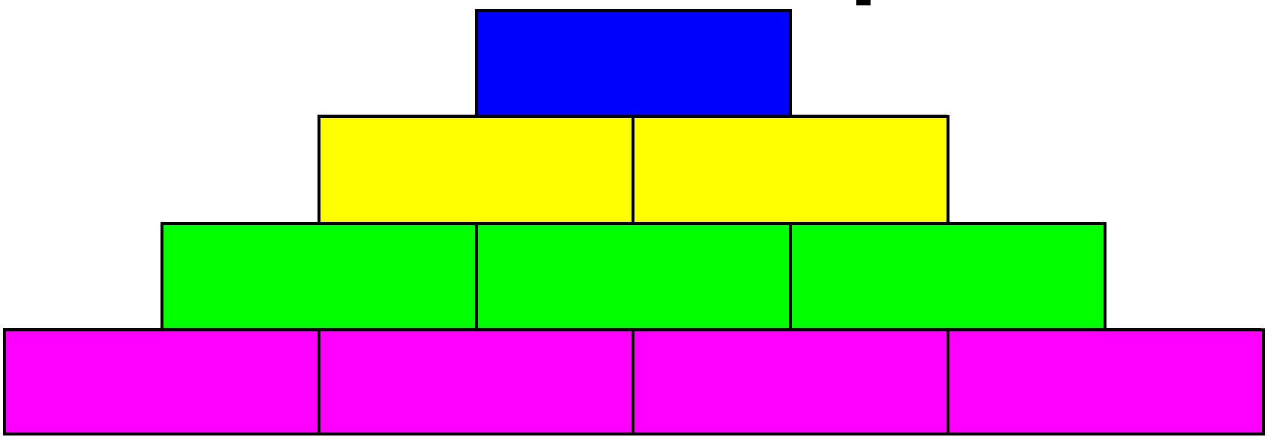
Each day the frog climbs 3m, but falls back 2m at night. How many days does it take for him to escape?

Number Pyramids ☆☆ 7

Can you work out what number will be at the top of the pyramid?



Can you make a pyramid with 100 at the top?



Number Race ☆☆ 8

Here are the first few square numbers:

1	4	9	16	25	36	49

Here are the first few Fibonacci numbers:

1	1	2	3	5	8	13

The square numbers are well in the lead. Do the Fibonacci numbers ever catch up?

...and here are the triangular numbers coming up on the rails...

1	3	6	10	15	21	28

Perfection ☆☆☆ 9

6 is a very special number.

The factors of 6 are 1, 2, 3 and 6.

If we add the factors other than 6 we get 1+2+3=6.

Can you find another perfect number?

Magic Squares ☆☆☆ 10

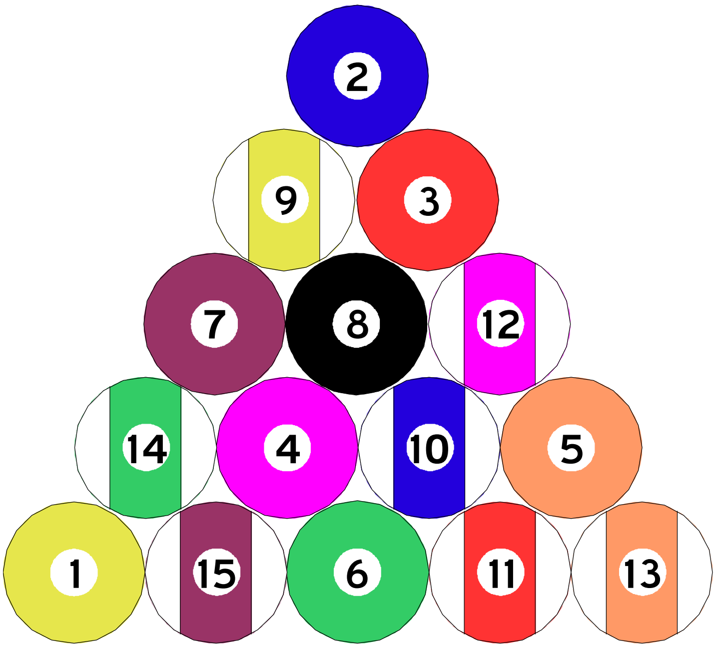
Can you put the digits 1 to 9 in a square so that every row, column and diagonal add to 15?

This example doesn’t work:

1	3	5	→9
9	6	4	→19
2	7	8	→17

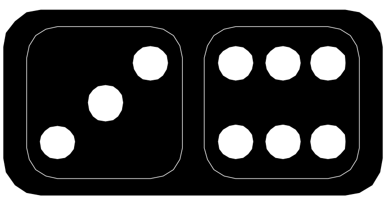
↙13 ↓12 ↓16 ↓17 ↘15

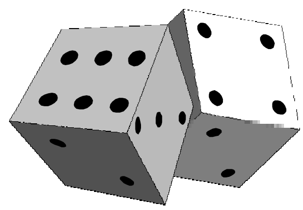
Pool balls are numbered from 1 to 15. What is the total of the numbers on all the balls?



Dominoes have two parts, each can have from 0 to 6 spots. How many different dominoes are there in a set?

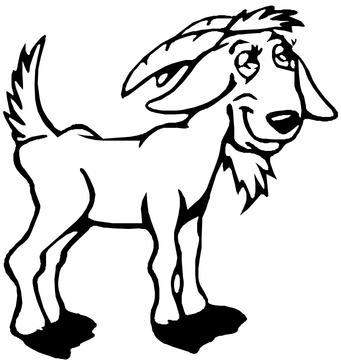
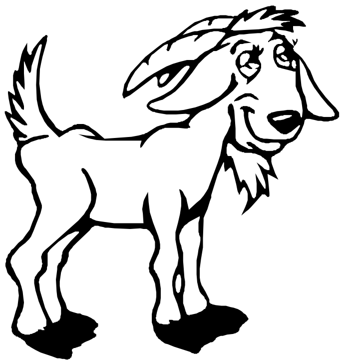
If you roll 2 dice and add the spots are you more likely to get an even or an odd number? What if you multiply instead?





Monty Hall ☆☆☆ 12

You are the winner in a quiz show and can choose a prize from behind 3 locked doors. Behind one door is a new car. Behind the other two doors are goats.

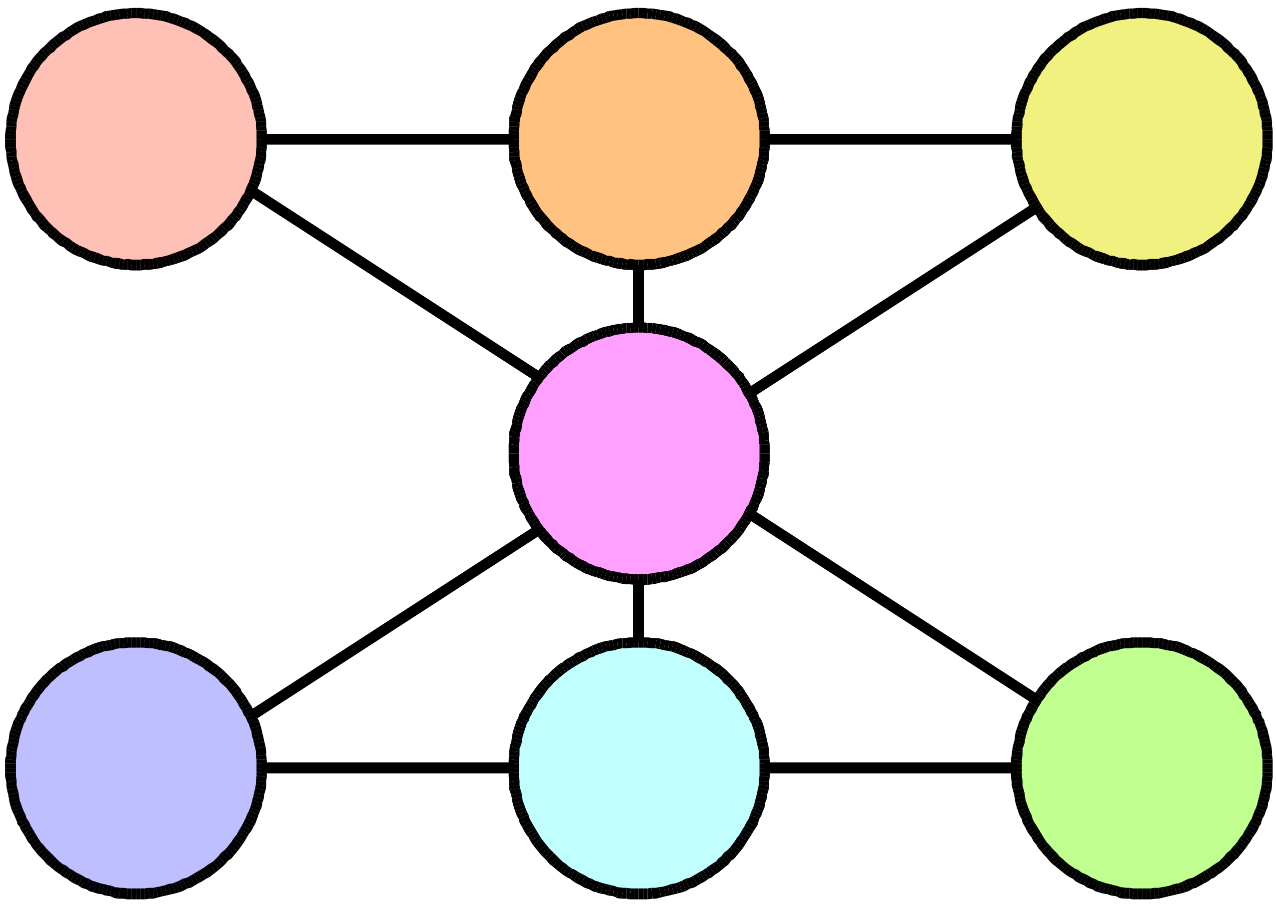




When you have made your choice the host opens one of the other doors to reveal a goat.

Should you stick with your choice, or switch to the other one? Or does it make no difference?

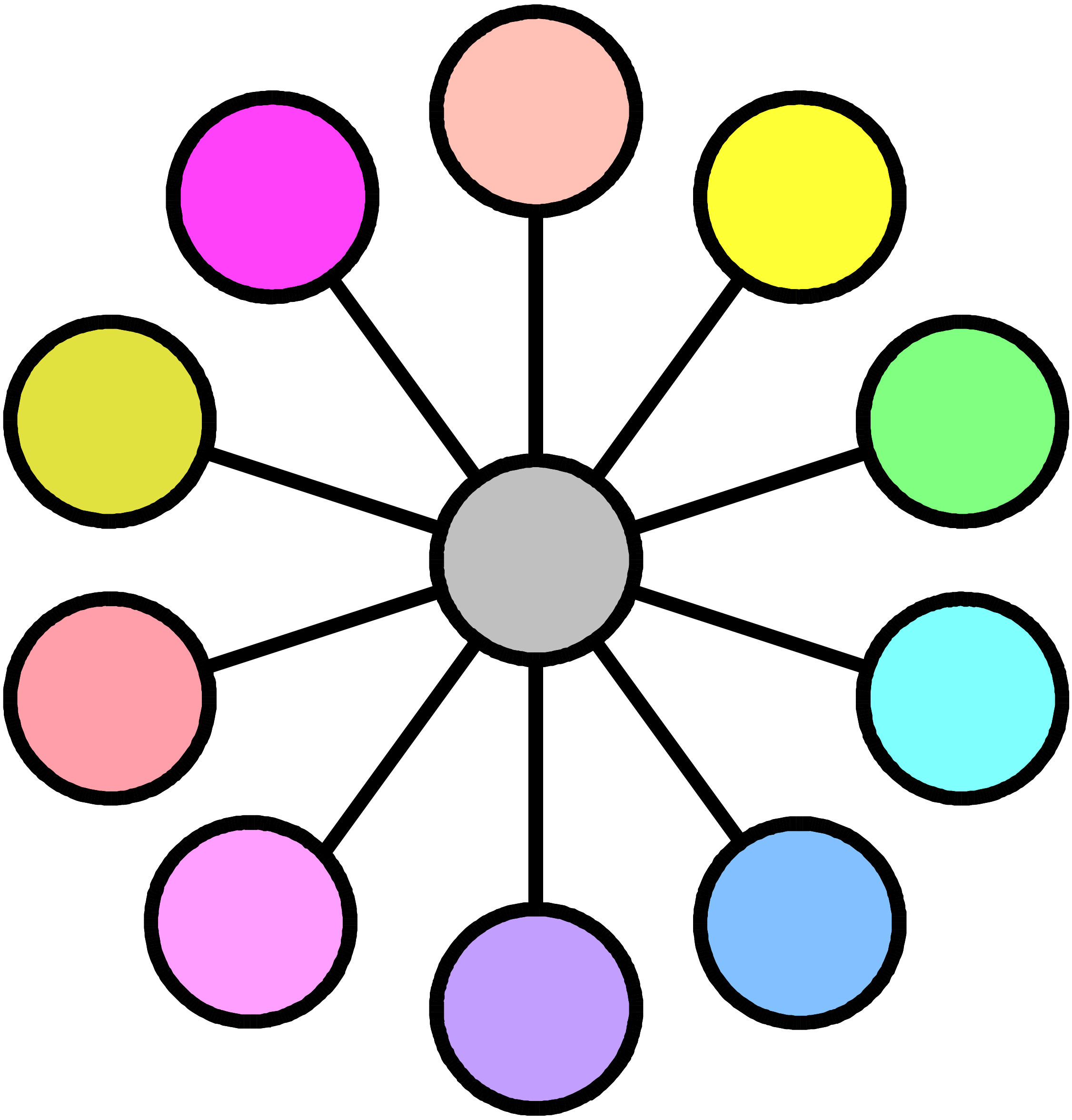
Number Lines 1 ☆☆ 13

Can you put the numbers 1 to 7 in each circle so that the total of every line is 12?



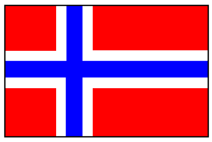
Number Lines 2 ☆ 14

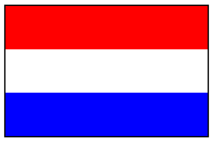
Can you put the digits 1 to 11 in the circles so that every line has the same total?

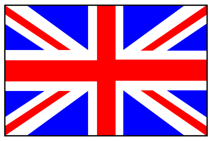


Words & Numbers ☆☆☆ 15

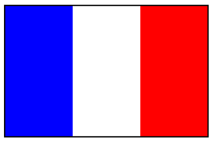
The number 4 in...

Norwegian,  **fire**

Dutch,  **vier**

and English  **four**

but not in

French  **quatre**

have something in common. They all have the same number of letters as their name (4).

Can you find more numbers like this?

Four Twos Make Ten ☆☆ 16

There are lots of ways of making 8 with four 2s using standard mathematical notation. For example:

$2+2+2+2$, or

even $2^{2+2} \div 2$.

Can you find another way?

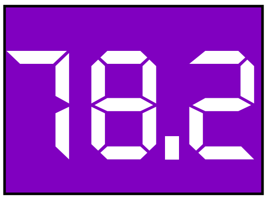
Can you find a way of making 9 with four 2s?

What is the biggest number you can make with four 2s?

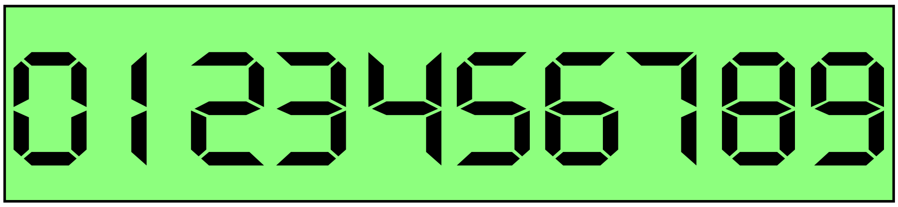
What is the smallest number you can't make with four 2s?

Petrol Prices ☆☆ 17

Some petrol stations display prices by sticking segments together to make numbers.



What is the largest number you can make with 10 segments?

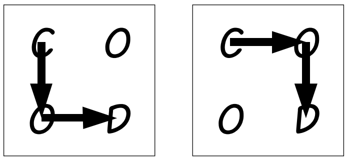


What is the largest you can make with 16 segments?

A Fishy Tale ☆☆☆ 18

If you write the word COD in a grid like this there are two ways of spelling the word.

C	O
O	D



How many ways can you make HAKE?

H	A	K
A	K	E

What about COLEY or SALMON?

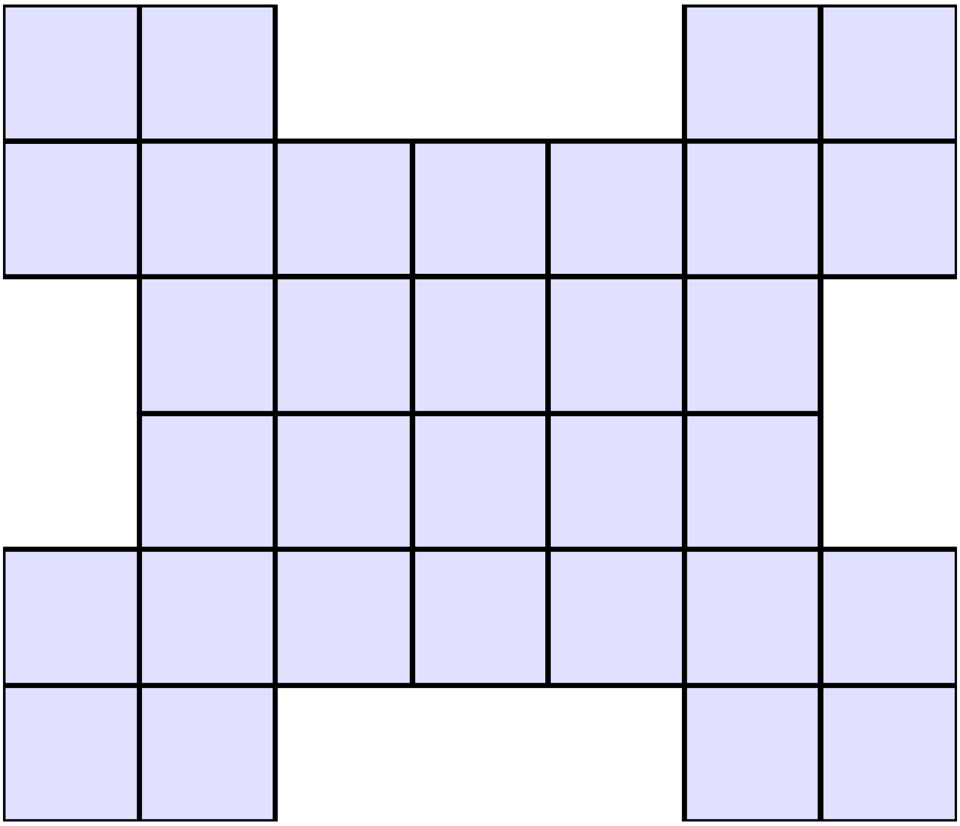
C	O	L
O	L	E
L	E	Y

S	A	L	M
A	L	M	O
L	M	O	N

Can you predict how many ways there are to make BARRACUDA?

How Many Squares? ☆☆ 19

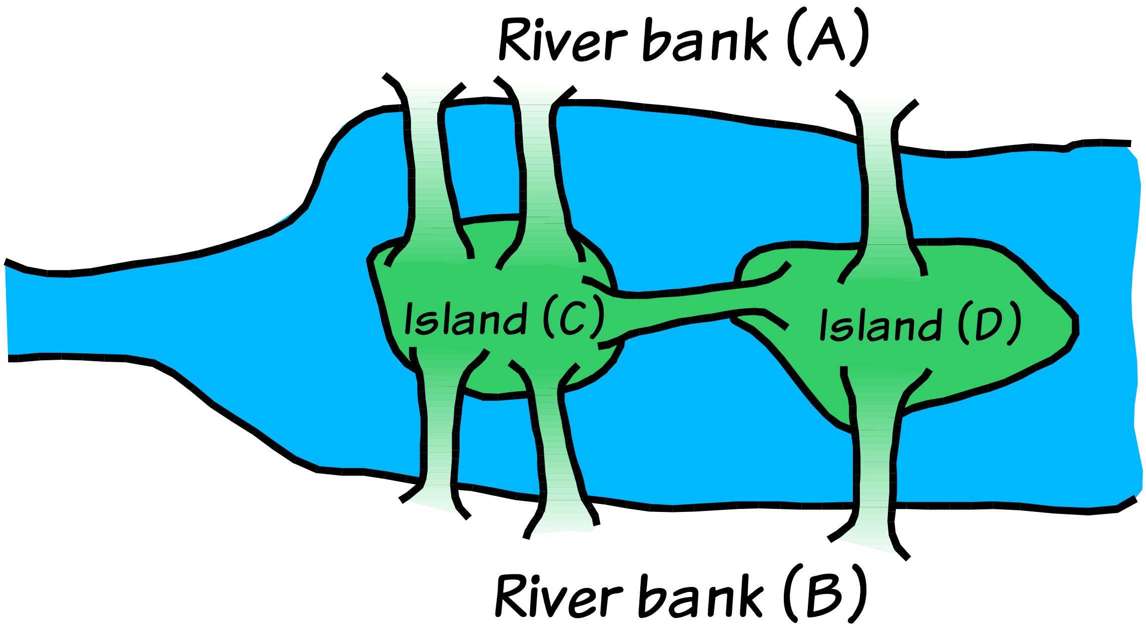
How many squares can you see in this pattern?



How many rectangles are there?

The Königsberg Bridges ☆ 20

The city of Königsberg had seven bridges that crossed the river Pregel.



Can you find a way of crossing all the bridges exactly once?

You can't go over a bridge more than once.

Crooked Calculator ☆☆ 21

Eric has got all the sums wrong. Each time he pressed exactly one wrong key.

ON/C 2 5 + 3 6 =	900
ON/C 5 + 3 + 2 =	317
ON/C 8 + 8 + 2 =	3
ON/C √ 1 6 + 1 =	41

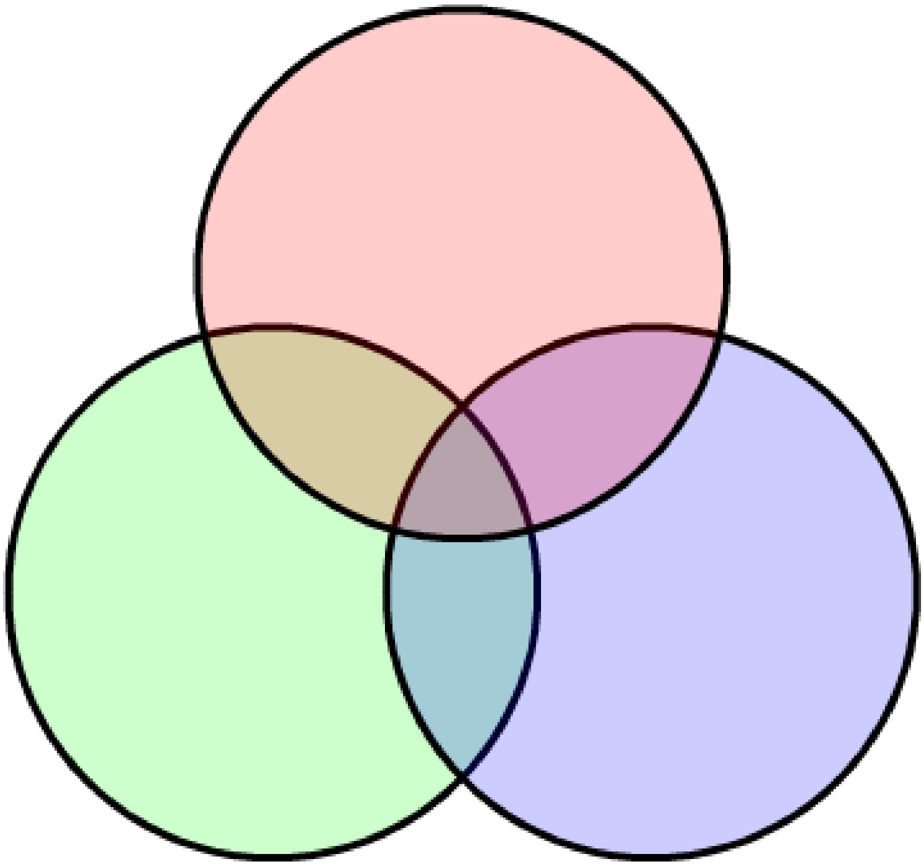
Can you work out which keys he actually pressed?

Eric manages to press the right keys but gets them in the wrong order. Can you get the keys in the right order?

ON/C 2 3 + 5 6 =	61
ON/C 5 + 3 − 2 =	0
ON/C 8 ÷ 8 4 2 =	103
ON/C √ 3 1 + 6 =	7

In the town of Ketterby

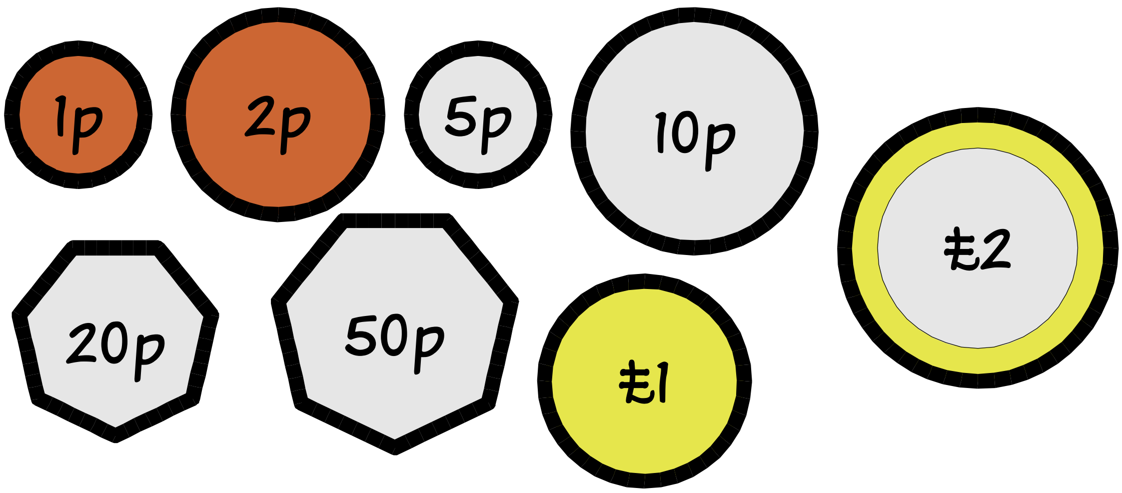
80% drink cola
70% drink coffee
50% drink tea



Is it certain there is someone who drinks cola, coffee and tea?

Coin of the Realm ☆ 23

These are the coins commonly used in Britain today

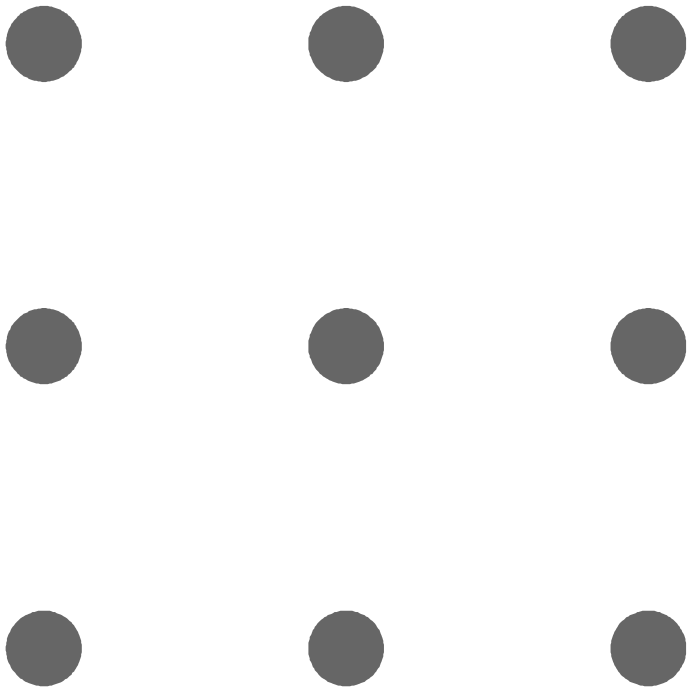


What is the fewest number of coins you need to make (a) 83p (b) £1.34 (c) £5.27?

What is the smallest amount that needs more than 5 coins?

Four Lines ☆☆ 24

Put 9 dots in a square like this



Can you go through all 9 dots with four straight lines?

You can't take your pen off the paper.
You can start where you like.

Back To Front ☆ 25

Write down a 2 digit number

62

Reverse the digits

26

Work out the difference

62 − 26 = 36

Try more 2 digit numbers. Can you see a pattern?

What happens with 3 digit numbers?

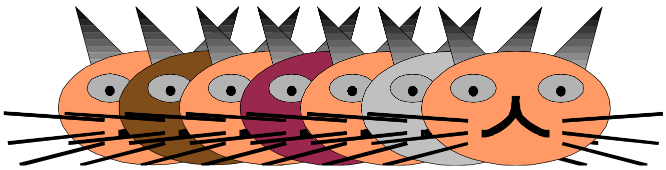
St. Ives ☆ 26

As I was going to St. Ives,

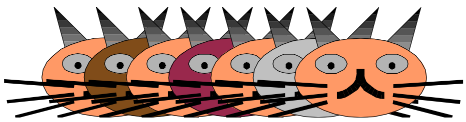
I met a man with 7 wives.

Each of the wives had 7 sacks.

In each sack were 7 cats.



Each of the cats had 7 kits.

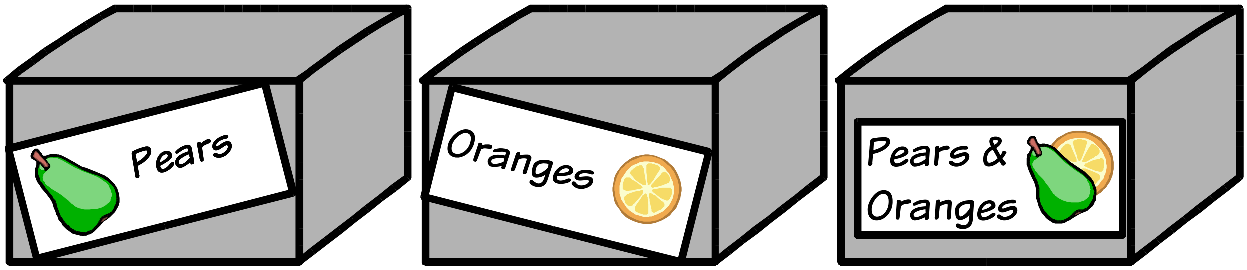


Kits, cats, sacks, wives – how many were going to St. Ives?

Fruitful ☆☆☆ 27

There are three boxes.

One box contains pears, one contains oranges, and one contains pears and oranges.

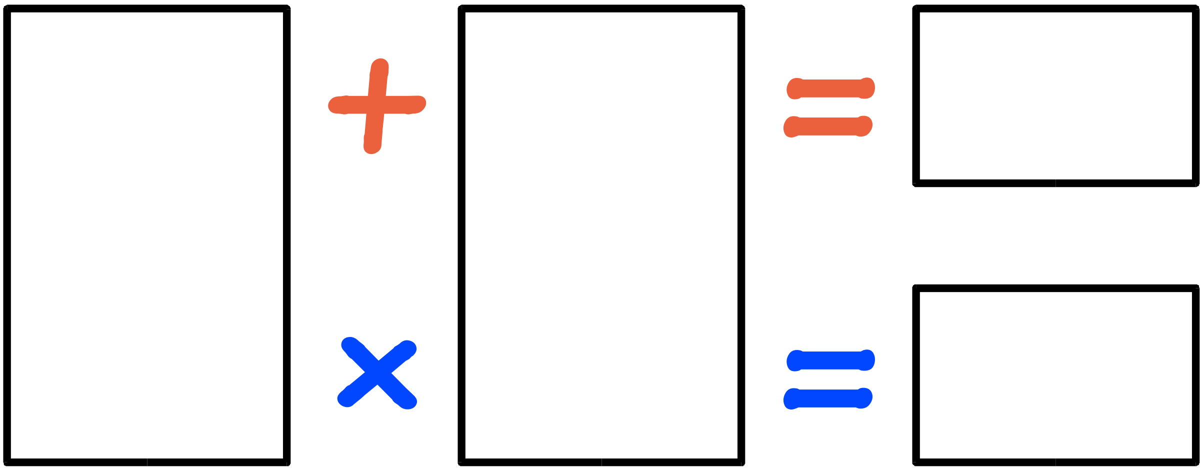


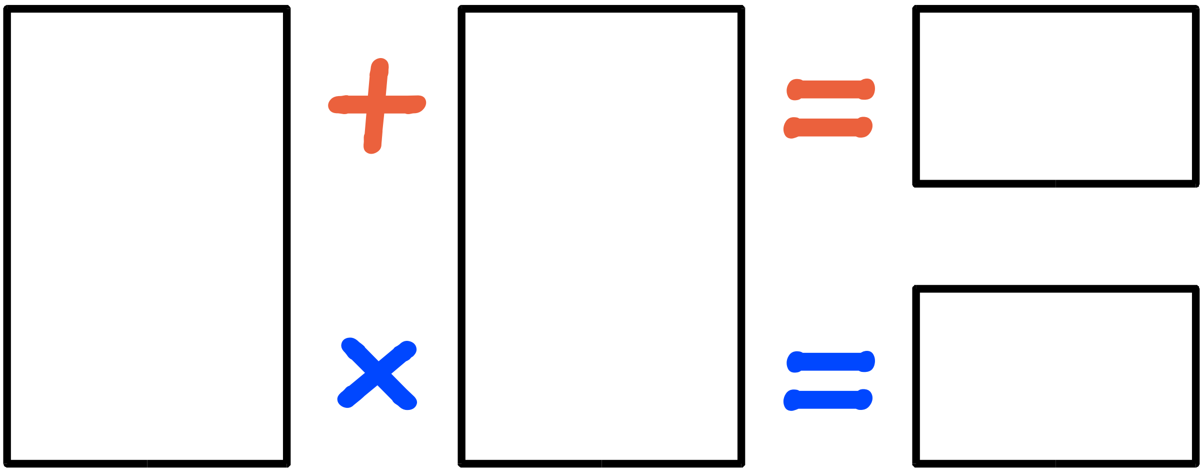
The labels have fallen off and all have been stuck back on the wrong boxes.

Barry opens one box and without looking in the box takes out one piece of fruit. He looks at the fruit and immediately puts the labels on the the right boxes.

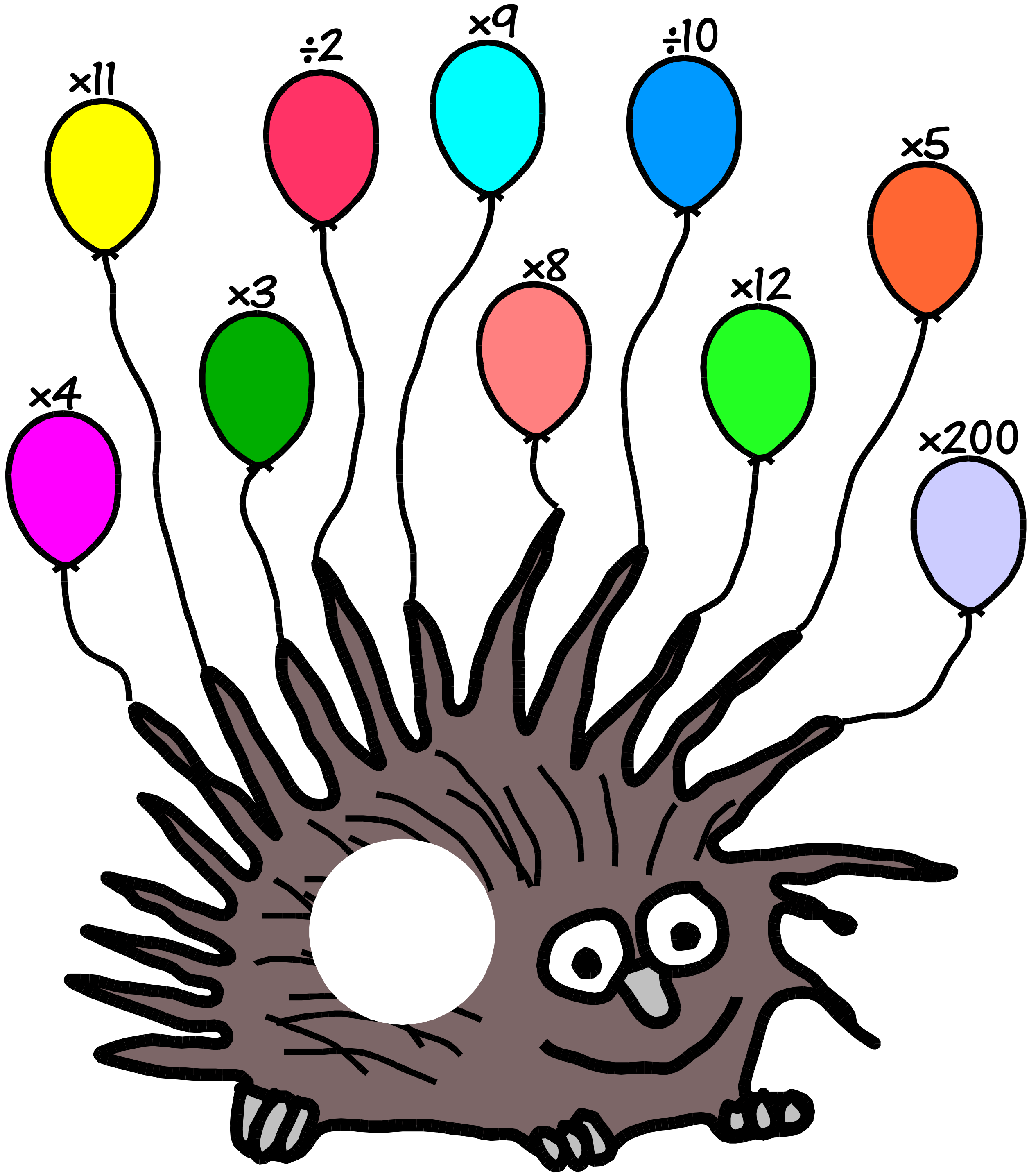
How did he do it?

Find 2 numbers whose sum is _____ and product is _____ .





Harry The Hedgehog A2



Today's Number A3

Today's number is

Add 17	
Double it	
Multiply it by 10	
Halve it	
Subtract 7	
Multiply by 6	
Square it	
Find its factors	
Find ¼ of it	

Notes

1. Find The Path

Here is one solution:
5 + 7 + 6 + 6 + 9 + 4 + 5 + 7 + 4 = 53

4	9	7	7	4
8	9	4	5	7
6	6	4	9	9
7	8	8	8	6
5	5	6	5	5

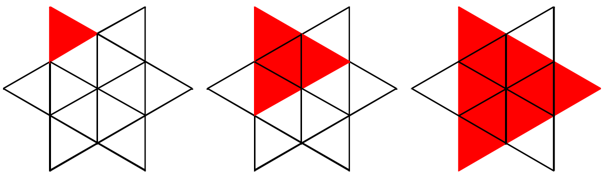
A tougher challenge is to find a route to make 60.

2. Missing Number

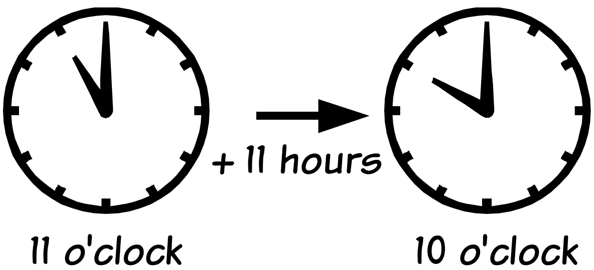
The missing number is **5** (add the bottom two numbers and divide by the top number). Students may be encouraged to come up with puzzles of their own.

3. How Many Triangles?

12 + 6 + 2 = **20**

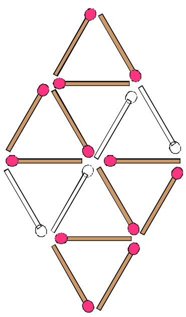


4. Times Table



11 + 11 = 10. This could clearly be extended to other moduli. The more observant students may spot the clue in the title.

5. Missing Matches



How may different solutions are there?

6. The Jumping Frog

It will take him 28 days to escape. After 27 days and nights the frog has only 3 metres to go. On the 28th day the frog is able to jump clear.

7. Number Pyramids

57. Each number is obtained by adding the two immediately below it. A good strategy for solving these puzzles is to start with the target number in place and to work downwards. This can clearly be extended to non-integral and negative targets.

8. Number Race

The 12th square and Fibonacci numbers are both 144. After this the Fibonacci numbers are in the lead. The 10th Fibonacci and triangular numbers are both 55. After this the triangular numbers are forever doomed to 3rd place. A good opportunity to explore the Fibonacci numbers.

9. Perfection

28 = 1 + 2 + 4 + 7 + 14. The next two are **496** and **8128**. The even perfect numbers are related to the *Mersenne primes*.
$2^p - 1$ is a prime if and only if $2^{p-1}(2^p - 1)$ is a perfect number. It is not known if there are any odd perfect numbers. You could investigate multiple-perfect numbers, where the sum of the proper divisors of a number is an exact multiple of it.

10. Magic Squares

Here is one solution:

8	1	6	=15
3	5	7	=15
4	9	2	=15
=15	=15	=15	

Diagonals: 4 + 5 + 6 = 15, 8 + 5 + 2 =15.
See Appendix A for a fairly simple method for constructing magic squares of odd order.

11. Games

Pool: 120

Pair off the balls 1+15, 2+14, etc.
This gives seven pairs. 7 × 16 = 112.
Adding the 8 ball gives 120.

Dominoes: 28

List the dominoes with a blank (0):
0-0, 0-1, 0-2, ..., 0-6 (7 dominoes)
Then list those with a 1:
1-1, 1-2, 1-3, ..., 1-6 (6 dominoes).
We miss out the 1-0 as this is in the first list.
Continuing in the same way gives
1 + 2 + 3 + ... + 7 = 28

Dice: $\frac{1}{2}$

$$P(\text{odd on dice}) = P(\text{even}) = \frac{1}{2}$$

$$P(\text{odd sum}) = P(\text{odd + even}) + P(\text{even + odd})$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

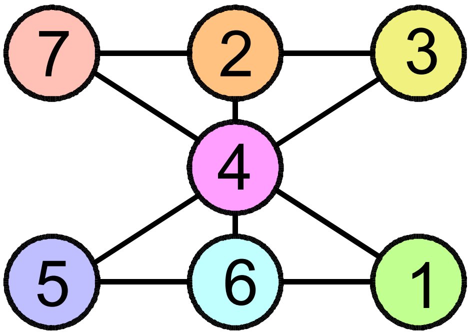
Alternatively you could draw a sample space diagram and get counting...

12. The Monty Hall Problem

You should **switch** (assuming you would prefer a car to a goat). It is tempting to assume that the two remaining doors have an equal chance of hiding the star prize, but this neglects the fact that the host's choice of door to open is not independent of your initial choice.
The probability of your initial choice being correct is $\frac{1}{3}$. This probability doesn't change when the host opens one of the other doors. So the probability that the remaining door hides the prize is $\frac{2}{3}$. You can find lots more on the 'Monty Hall Problem' at
http://math.rice.edu/~ddonovan/montyurl.html

13. Number Lines 1

You could give the hint that the centre digit is 4.



14. Number Lines 2

This is a little easier than the previous puzzle. Put 6 in the centre. Then place 1 opposite 11, 2 opposite 10, etc. to give totals of 18. In both puzzles the realisation that you should put the median number in the centre to maintain symmetry helps enormously. (You could also put 1 or 11 in the centre, since the remaining numbers can be paired off in these cases to give totals of 14 and 22.

15. Words & Numbers

An *orthonymic number* is one that has the same number of letters as its name suggests. "Four" is *probably* unique in English. (How would you prove it?) Some more exotic examples include "queig" (Manx, 5); "bederatzi" (Basque, 9); "du" (Esperanto, 2) and "amashumi amabili nesikhombisa" (Zulu, 27).
An interesting question is to find expressions that are orthonymic, e.g. "five add seven" (12 letters).

16. Four Twos Make Ten

Two other ways of making 8 are 2×2 + 2×2 and the less obvious $(\frac{2}{2}+2)!+2$.
$2^{2^{2+2}} = 65536$ *may* be the largest if we restrict ourselves to powers and the 4 basic operations. 7 is the smallest positive integer that cannot be made without resource to factorials etc. A calculator is useful.

17. Petrol Prices

11111 and 11111111 — a good way of showing the importance of place value. A more interesting question is to ask how many *different* numbers can be made with 7 segments.

18. A Fishy Tale

COD: 2, HAKE: 3, COLEY: 6, SALMON: 10

The formula for a word of length n is

$$\begin{cases} \dfrac{(n-1)!}{(\frac{n-1}{2})!(\frac{n-1}{2})!} & \text{, if } n \text{ is odd} \\[2ex] \dfrac{(n-1)!}{(\frac{n}{2}-1)!(\frac{n}{2})!} & \text{, if } n \text{ is even} \end{cases}$$

So BARRACUDA gives 8!/(4!4!) = 70.

This is discussed in Appendix B.

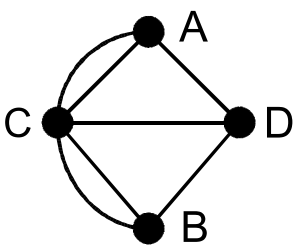
19. How Many Squares?

28 @ 1×1, 16 @ 2×2, 6 @ 3×3, 2 @ 4×4.

Total: **52**

20. The Bridges Of Königsberg

It can't be done. Look at this network:



Every time we enter a piece of land we must leave it by a different bridge. So there must be an even number of bridges attached to each piece of land (except for the start and finish). There are four pieces of land with an odd number of bridges. Even if we take the start and finish into account there must be another piece of land with an unused bridge. If you remove one of the bridges then it becomes possible (such a path is called an *Eulerian path*). Does it matter which bridge you remove?

You can find details of a visit by a mathematician to Königsberg at http://www.amt.canberra.edu.au/koenigs.html

21. Crooked Calculator

ON/C 2 5 × 3 6 = 900
ON/C 5 + 3 1 2 = 317
ON/C 8 ÷ 8 + 2 = 3
ON/C √ 1 6 8 1 = 41

ON/C 2 5 + 3 6 = 61
ON/C 3 + 2 − 5 = 0
ON/C 8 2 4 ÷ 8 = 103
ON/C √ 3 6 + 1 = 7

22. Drinkers

No – it is not certain. Suppose there were 10 people in Ketterby. You could have:

cola	✓	✓	✓	✓	✓	✓	✓	✓		
coffee				✓	✓	✓	✓	✓	✓	✓
tea	✓	✓	✓						✓	✓

20% is the maximum possible percentage of people who drink none of the beverages.

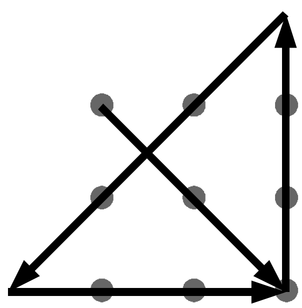
23. Coin of the Realm

(a) 83p = 50p + 20p + 10p + 2p + 1p
(b) £1.34 = £1 + 20p + 10p + 2p + 2p
(c) £5.27 = £2 + £2 + £1 + 20p + 5p + 2p
88p = 50p + 20p + 10p + 5p + 2p + 1p needs six coins.

24. Four Lines

You have to (literally) think outside the box to solve this old chestnut.

Here's one solution:



25. Back To Front

The answers are all **multiples of 9**.
It's not too difficult to prove this:
The original number can be written as $x=10a+b$ where a and b are the two digits. The reversed number is therefore $y=10b+a$ and the difference is

$$|x-y|=|(10a+b)-(10b+a)|=|9a-9b| =9|a-b|$$

i.e. 9 times the difference in the digits of the original number.
For three digit numbers you will get **multiples of 99.**

26. St. Ives

A trick question. Since you met the man on the way to St. Ives, he and his entourage must be coming *from* St. Ives. So the answer is just one – you. If you're feeling evil you might want to conspicuously distribute calculators for this puzzle.
If you do actually go through the calculations you should get $7+7^2+7^3+7^4$ = 7 + 49 + 343 + 2401 = 2800.
This problem is very old indeed. It is supposed to date back to the ancient Egyptians:
http://mathsforeurope.digibel.be/story.htm

27. Fruitful

Barry opened the box labelled "pears & oranges". This box must contain only pears or only oranges. If he picks a pear then he knows the box he opens is "pears" and the other two boxes must be "oranges" and "pears and oranges". The one labelled "oranges" must be wrong so it is labelled as "pears and oranges". A similar argument works if he picks an orange.
In summary:

He picks	*Pears*	*Oranges*	*P & O*
A pear	Oranges	P & O	Pears
An orange	P & O	Pears	Oranges

A1. Sum and Product

Here are a few suggestions:

sum	*product*	*numbers*
10	9	1, 9
15	50	5, 10
8	15	3, 5
18	56	4, 14
8½	4	½, 8
4	3¾	1½, 2½
3	1.89	0.9, 2.1

A little algebra shows that for sum s and product p the numbers are $\frac{1}{2}(s\pm\sqrt{s^2-4p})$.

A2. Harry The Hedgehog

Write a number in his belly...

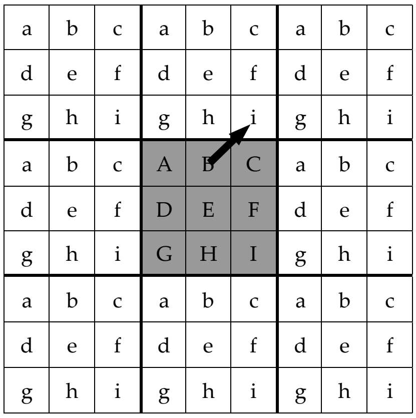
A3. Today's Number

Just for fun try π.

Appendix A – Magic Squares

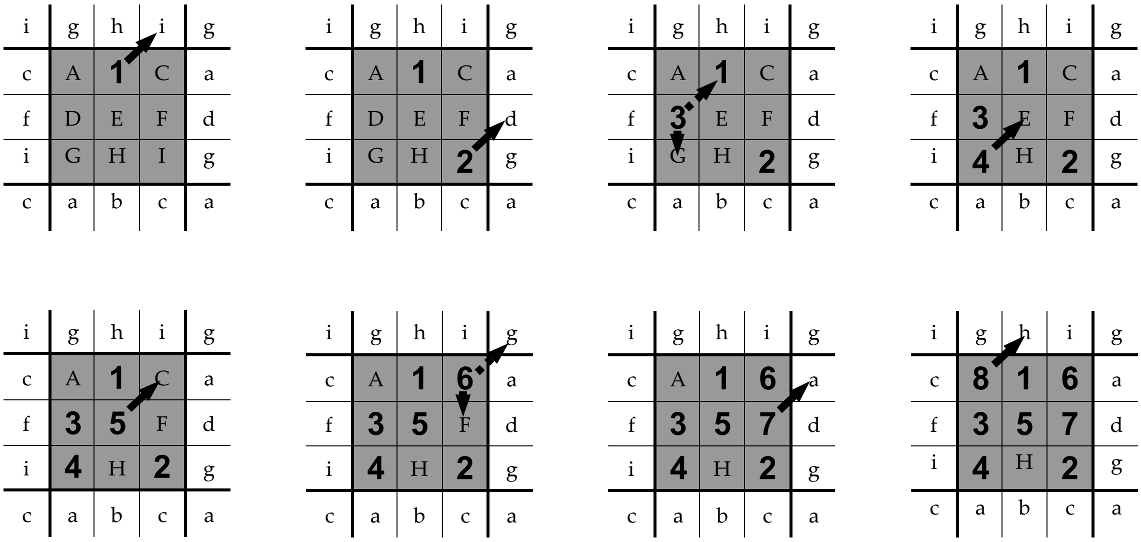
There is a well known way of constructing magic squares of **odd** order: 3×3, 5×5, etc.

1. Put the number 1 in the top centre space.
2. Move diagonally up and right. If that space is already occupied then move down instead. Put the next consecutive number in that space.
3. Repeat until all squares are filled.



Of course you will find that your moves take you out of the original square. It helps to think of the plane being tiled with copies of your square, so that you move to the corresponding space. In the diagram to the right a move from *B* takes you to *i* (the bottom right space), so the next number will go in *I*, the corresponding space in the main square.

Here's how it works for a 3×3 square:



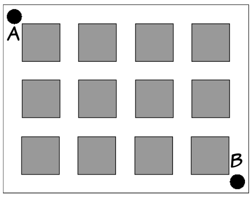
8	1	6
3	5	7
4	9	2

After we have placed 3 we find that the slot for 4 is already filled,so we move down instead. We also have to do this when trying to place 7.
See if you can reproduce this 5×5 magic square using this method.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

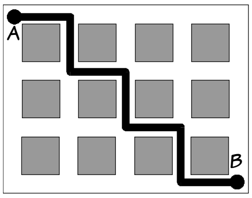
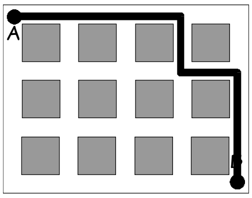
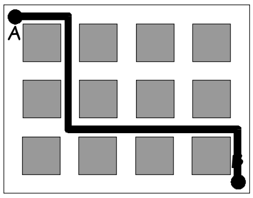
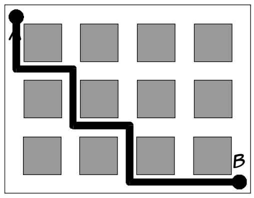
Appendix B – City Blocks

Puzzle 18 is in fact a variation of the 'City Blocks Problem':



"You are in a city where the roads are laid out in a grid. At each intersection you can only travel East or South. How many different ways are there of getting from A to B?"

You should be able to convince yourself that whatever route you choose requires four 'Easts' and three 'Souths':

			
ESESESE	EEESESS	ESSEEES	SESESEE

So the problem reduces to "How many different ways are there of writing four E's and three S's?" It's not immediately obvious how we can find the answer. So we look at a similar problem:

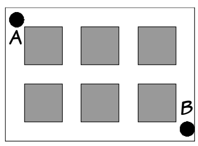
"How many different *permutations* are there of the letters A, B, C, D, E, F, G."

There are seven ways of choosing the first letter.
No matter which letter you choose (e.g. C) there are six letters remaining (e.g. A, B, D, E, F, G) , so there are 7 × 6 ways of picking the first two letters.
Now there are five letters left so we have 7 × 6 × 5 ways of picking the first three letters.
Continuing in this way we find there are 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040 permutations.

We write 7 × 6 × 5 × 4 × 3 × 2 × 1 as 7! ("factorial seven").

The number of permutations of n different objects is $n!$.

That's fine if all the letters are different but in our City Block problem some of the letters are repeated. Let's look at a simpler problem:



Here we have three 'Easts' and two 'Souths'. So the problem is the same as "How many permutations are there of the letters E, E, E, S, S ?"

Appendix B – City Blocks

A bit of trial and error should convince you there are ten permutations:

EEESS EESES EESSE ESEES ESESE ESSEE SEEES SEESE SESEE SSEEE

We know that there are 5! = 5 × 4 × 3 × 2 × 1 = 120 permutations of A, B, C, D, E. So by repeating letters we have drastically reduced the number of permutations. We can see why by distinguishing the E's. Suppose we write the E's as E_1, E_2, E_3. We can see that each of our solutions above can be written in six ways, e.g. The solution ESSEE would become six different solutions:

$E_1SSE_2E_3$ $E_1SSE_3E_2$ $E_2SSE_1E_3$ $E_2SSE_3E_1$ $E_3SSE_1E_2$ $E_3SSE_2E_1$

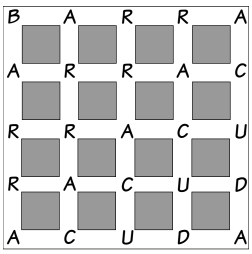
There are in fact 3! = 3 × 2 × 1 = 6 different permutations of E_1, E_2, E_3. The number of permutations is reduced by a factor of 3!. Similarly repeating the S reduces the number of permutations by a factor of 2!. So for our smaller City Block problem we have $\frac{5!}{3!2!}=\frac{120}{6\times 2}=10$ solutions.

We have a general solution for any City Block problem:

> A City Block problem with e 'Easts' and s 'Souths' has $\frac{(e+s)!}{e!s!}$ different routes.

Our original city block problem has $e=4$ and $s=3$ so has $\frac{7!}{4!3!}=\frac{5040}{24\times 6}=35$ different routes.

We can apply this to the 'Fishy Tale' problem. Here's BARRACUDA written as a City Block problem:



Here $e=4$ and $s=4$ so there are $\frac{8!}{4!4!}=\frac{40320}{24\times 24}=70$ routes.

A little more work leads to the formula given in the notes:

A word of length n can be spelled in $\begin{cases} \dfrac{(n-1)!}{(\frac{n-1}{2})!(\frac{n-1}{2})!} & \text{, if } n \text{ is odd} \\ \dfrac{(n-1)!}{(\frac{n}{2}-1)!(\frac{n}{2})!} & \text{, if } n \text{ is even} \end{cases}$ ways.